B.Sc. - II (CBCS Pattern) Semester-IV

USMT-08 - Mathematics-II Paper-VIII - Elementary Number Theory

P. Pages : 2 | GUG/S/25/12015(S)

Time : Three Hours | * 4 7 4 1 * | Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that, the product of any m consecutive integers is divisible by m! **6**

b) Find all positive integers $n(<17)$ for which $n!+(n+1)!+(n+2)!$ is an integral multiple of 49. **6**

OR

c) Find the greatest common divisors of 275 and 200 and express it in the form 275x+200y. **6**

d) Prove that for positive integers a and b $(a,b)[a,b]=ab$. **6**

UNIT – II

2. a) Prove that, there are an infinite number of primes. **6**

b) If P is a prime and $P/a_1 a_2......a_n$, then prove that P divides at least one factor a_i of the product i.e. P/a_i, for some i, where $1 \le i \le n$. **6**

OR

c) Prove that, there are infinitely many primes of the form 4n+3, where n is a positive integer. **6**

d) If (a,b) = 1 then show that $(a^2, b^2) = 1$ and prove that $(a^2, b^2) = c^2$, if (a, b) = c, c > 0. **6**

UNIT – III

3. a) Prove that, congruence is an equivalence relation. **6**

b) Let $a_1, a_2, b_1, b_2 \in Z$ such that $a_1 \equiv b_1$ (mod m) and $a_2 \equiv b_2$ (mod m) then prove that **6**

i) $(a_1 + a_2) \equiv (b_1 + b_2)$ (mod m)

ii) $a_1 a_2 \equiv b_1 b_2$ (mod m)

OR

c) Prove that, a number a has an inverse modulo m if and only if (a, m) = 1. **6**

d) Find all solutions of $15x \equiv 12$ (mod g). **6**

UNIT – IV

4. a) Let P be a prime and K a positive integer then prove that $\phi\left(p^k\right) = p^k - p^{k-1} = p^k\left(1 - \frac{1}{p}\right)$. **6**

b) Find all the positive integers x and y such that $x^{\phi(y)} = y$. **6**

OR

c) Find all the positive integers a and b which satisfy the equation $\phi(ab) = \phi(a) + \phi(b)$. **6**

d) Prove that, the Mobius μ - function is multiplicative. **6**

5. Solve **any six** questions.

a) Show that, if a is an integer then 3 divides $a^3 - a$ **2**

b) Prove that If $(a,b) = d$ then $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$ **2**

c) If P is a prime and P/ab then prove that P/a or P/b where a and b are integers. **2**

d) If a prime P>3 then show that 2P+1 and 4P+1 cannot be prime simultaneously. **2**

e) Show that if a, b, m and n are integers such that m>0, n>0, m/n and $a \equiv b$ (mod n) then $a \equiv b$ (mod m). **2**

f) For which positive integers m, is the following statement true, $100 \equiv 1 \pmod{m}$, $m > 1$. **2**

g) If P is a positive integer with $\phi(P) = P - 1$ then prove that P is prime. **2**

h) Solve $ax \equiv b$ (mod m) with (a, m) = 1. **2**

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